Reg.No. ____________

Karunya UNIVERSITY

(Karunya Institute of Technology & Sciences)

(Declared as Deemed-to-be University under Sec.3 of the UGC Act, 1956)

End Semester Examination – Nov/Dec – 2016

			Semester	**:**	**2016-17 ODD**
Code	**:**	**15MA3004**	**Duration**	**:**	**3hrs**
Sub. Name :		**Real Analysis**	**Max. marks :**		**100**

ANSWER ALL QUESTIONS (5 x 20 = 100 Marks)

Q. No.	Sub Div.	Questions	Course Outcome	Marks
1.	a.	If $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + ... + \frac{x^n}{n!} + ...$, then show that the number 'e' is irrational.	CO1	8
	b.	If $a \geq 0$, then prove that $\|x\| \leq a$ if and only if $-a \leq x \leq a$.	CO1	6
	c.	For arbitrary real *x* and *y*, show that $\|x + y\| \leq \|x\| + \|y\|$.	CO1	6
		(OR)		
2.	a.	State and prove the unique factorization theorem.	CO1	10
	b.	State and prove Cauchy Schwarz inequality.	CO1	10
3.	a.	Prove that the set of all real numbers is uncountable.	CO1	10
	b.	Let Z^+ denote the set of all positive integers. Then show that the Cartesian product $Z^+ \times Z^+$ is countable.	CO1	10
		(OR)		
4.	a.	Define converse of a relation. Also prove that if the function *F* is one-to-one on its domain, then $\check{F}$ is also a function.	CO1	8
	b.	Define countable and uncountable sets. Also prove that every subset of a countable set is countable.	CO1	12
5.	a.	Define open set and prove that the intersection of finite collection of open sets is open	CO2	8
	b.	State and prove Heine-Borel covering theorem.	CO2	12
		(OR)		
6.	a.	Let $\bar{x}$ and $\bar{y}$ denote points in R^n. Then prove that (i) $\\|\bar{x}\\| \geq 0$, and $\\|\bar{x}\\| = 0$ if and only if $\bar{x} = 0$. (ii) $\\|a\bar{x}\\| = \|a\| \\|\bar{x}\\|$ for every real a. (iii) $\\|\bar{x} - \bar{y}\\| = \\|\bar{y} - \bar{x}\\|$. (iv) $\|\bar{x}.\bar{y}\| \leq \\|\bar{x}\\| \\|\bar{y}\\|$ (v) $\\|\bar{x} + \bar{y}\\| \leq \\|\bar{x}\\| \\|\bar{y}\\|$	CO2	10
	b.	If $A \subseteq R^n$ and let *F* be an open covering of *A*, then prove that there is a countable sub collection of *F* which also covers *A*.	CO2	10
7.	a.	Let *f* be a function defined on *(a, b)*, then prove that there is the function f^* which is continuous at '*c*'and which satisfies $f(x) - f(c) = (x - c) f^*(x)$, for all *x* in *(a, b)*, with $f'(c) = f^*(c)$. Conversely, if there is a function f^*, continuous at *c*, then prove that *f* is differentiable at *c* and $f'(c) = f^*(c)$.	CO3	10

	b.	State and prove Mean-Value Theorem.	**CO3**	**10**
		(OR)		
8.	a.	Let $f: S \to T$ be a function from one metric space (S, d_S) to another (T, d_T) and if $p \in S$, prove that *f* is continuous at *p* if and only if, for every sequence $\{x_n\}$ in *S* converges to *p*, the sequence $\{f(x_n)\}$ in *T* converges to *f (p)*.	**CO3**	**10**
	b.	Let *f* and *g* are defined on *(a, b)* and differentiable at *c*. Prove that $f + g$, $f - g$ and $f \cdot g$ are differentiable at *c*. And also f/g is differentiable at *c* if $g(c) \neq 0$.	**CO3**	**10**
		Compulsory:		
9.	a.	State and prove Weierstrass M-test for the series of the function.	**CO3**	**10**
	b.	State and prove Dirichlet's test for uniform convergence of a series.	**CO3**	**10**

ALL THE BEST